C4 JUNE 09

1) $f(x) = (4+x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}}\left(1+\frac{x}{4}\right)^{-\frac{1}{2}}$

$= \frac{1}{2}\left(1 + \left(-\frac{1}{2}\right)\left(\frac{x}{4}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\left(\frac{x}{4}\right)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{6}\left(\frac{x}{4}\right)^3\right)$

$= \frac{1}{2} - \frac{1}{8}x + \frac{3}{128}x^2 - \frac{5}{1024}x^3$

2) $x = \frac{9\pi}{8}$ $\quad y = 3\cos\left(\frac{9\pi}{24}\right) = 1.14805$

Area $\simeq \frac{1}{2}\left(\frac{3\pi}{8}\right)\left(3 + 0 + 2\left(2.77164 + 2.12132 + 1.14805\right)\right)$

$= 8.884$

Area $= \int_0^{\frac{3\pi}{2}} 3\cos\left(\frac{x}{3}\right)dx = \left[-9\sin\left(\frac{x}{3}\right)\right]_0^{\frac{3\pi}{2}} = (-9) - (0)$

$= 9.$

3) $f(x) = \frac{A(x+1)(x+3) + B(2x+1)(x+3) + C(2x+1)(x+1)}{(2x+1)(x+1)(x+3)}$ $= 4 - 2x$

$x = -1 \Rightarrow -2B = 6 \Rightarrow B = -3.$

$x = -3 \Rightarrow 10C = 10 \Rightarrow C = 1$

$x = 0 \Rightarrow 3A + 3B + C = 4 \Rightarrow 3A = 12 \Rightarrow A = 4$

b) $\int f(x)dx = \int \frac{4}{2x+1}dx + \int \frac{-3}{x+1}dx + \int \frac{1}{x+3}dx$

$= 4 \times \frac{1}{2}\ln(2x+1) - 3\ln(x+1) + \ln(x+3) + c$

(ii) $\int_0^2 f(x)dx = (2\ln 5 - 3\ln 3 + \ln 5) - (2\ln 1 - 3\ln 1 + \ln 3)$

$= 3\ln 5 - 4\ln 3 = \ln 125 - \ln 81 = \ln\left(\frac{125}{81}\right)$

4) $\frac{d}{dx} ye^{-2x} = \frac{d}{dx}2x + \frac{d}{dx}y^2$

$u = y \quad v = e^{-2x}$

$u' = \frac{dy}{dx} \quad v' = -2e^{-2x}$

$\Rightarrow e^{-2x}\frac{dy}{dx} - 2e^{-2x}y = 2 + 2y\frac{dy}{dx}$

$\Rightarrow e^{-2x}\frac{dy}{dx} - 2y\frac{dy}{dx} = 2 + 2e^{-2x}y$

$\Rightarrow \frac{dy}{dx} = \frac{2+2e^{-2x}y}{e^{-2x}-2y}$

b) $(0,1)$ $M_t = \frac{2+2(1)}{1-2} = \frac{4}{-1} = -4 \Rightarrow M_n = \frac{1}{4}$

$y - 1 = \frac{1}{4}(x-0) \Rightarrow 4y - 4 = x \Rightarrow \underline{x - 4y + 4 = 0}$

5) $x = 2\cos 2t \qquad y = 6\sin t \qquad \frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$

$\frac{dx}{dx} = -4\sin 2t \qquad \frac{dy}{dt} = 6\cos t$

$\frac{dy}{dx} = \frac{-6\cos t}{4\sin 2t}$ at $t = \frac{\pi}{3}$ $M = \frac{-3}{2\sqrt{3}} = -\frac{3\sqrt{3}}{6} = \underline{-\frac{1}{2}\sqrt{3}}$

5b) $\frac{y}{6} = \sin t \qquad \frac{x}{2} = \cos 2t = 1 - 2\sin^2 t$

$\Rightarrow \frac{x}{2} = 1 - \frac{2y^2}{36} \Rightarrow x = 2 - \frac{y^2}{9} \Rightarrow \frac{y^2}{9} = 2 - x$

$\Rightarrow y^2 = 9(2-x) \Rightarrow y = \sqrt{9(2-x)} \Rightarrow y = 3\sqrt{2-x}$

$x = 2\cos 2t \Rightarrow -2 \leq x \leq 2 \Rightarrow \underline{k = 2}$

5c) $y \geq 0 \quad y \in \mathbb{R}.$

6) $\int \sqrt{5-x}\, dx \qquad u = (5-x)^{\frac{3}{2}} \qquad \times -\frac{2}{3}$

$= \int (5-x)^{\frac{1}{2}} dx \qquad \frac{du}{dx} = \frac{3}{2}(5-x)^{\frac{1}{2}} \times -1 \quad \times -\frac{2}{3}$

$= -\frac{2}{3}(5-x)^{\frac{3}{2}} + c$

b) $\int (x-1)\sqrt{5-x}\, dx \qquad u = (x-1) \qquad v = -\frac{2}{3}(5-x)^{\frac{3}{2}}$

$\frac{du}{dx} = 1 \qquad \frac{dv}{dx} = \sqrt{5-x}$

$= -\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - -\frac{2}{3}\int (5-x)^{\frac{3}{2}} dx$

$= -\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} + \frac{2}{3}\left(-\frac{2}{5}(5-x)^{\frac{5}{2}}\right) + c$

$= -\frac{2}{3}(x-1)(5-x)^{\frac{3}{2}} - \frac{4}{15}(5-x)^{\frac{5}{2}} + c$

c) $\int_1^5 (x-1)(\sqrt{5-x})\,dx = (0) - \left(0 - \frac{4}{15} \times 4^{\frac{5}{2}}\right) = \frac{4}{15} \times 32 = \frac{128}{15}$

7) $\overrightarrow{AB} = b - a = \begin{pmatrix} 10 \\ 14 \\ -4 \end{pmatrix} - \begin{pmatrix} 8 \\ 13 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$

$l = \begin{pmatrix} 8 \\ 13 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$

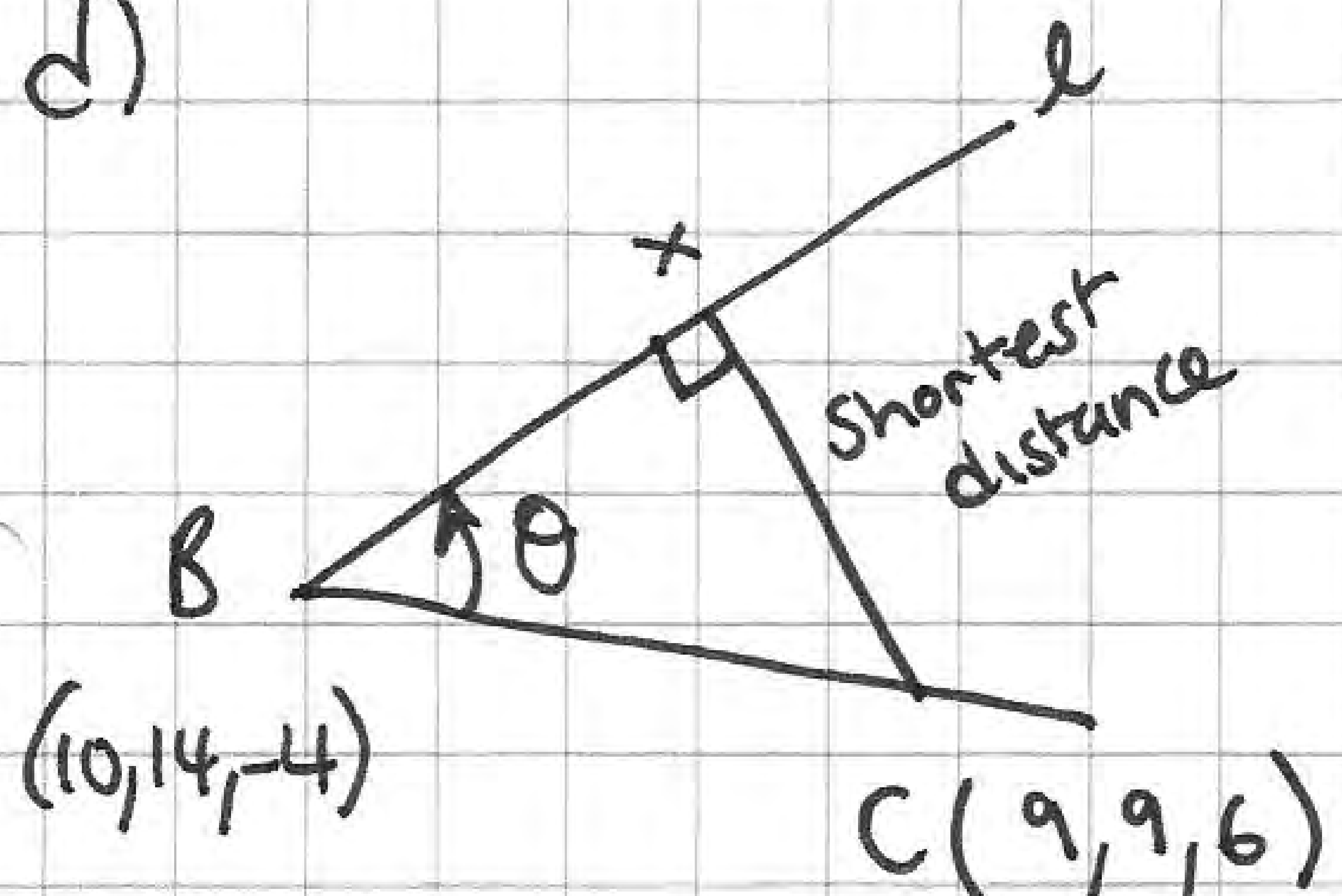
b) $\overrightarrow{CB} = b - c = \begin{pmatrix} 10 \\ 14 \\ -4 \end{pmatrix} - \begin{pmatrix} 9 \\ 9 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ -10 \end{pmatrix}$

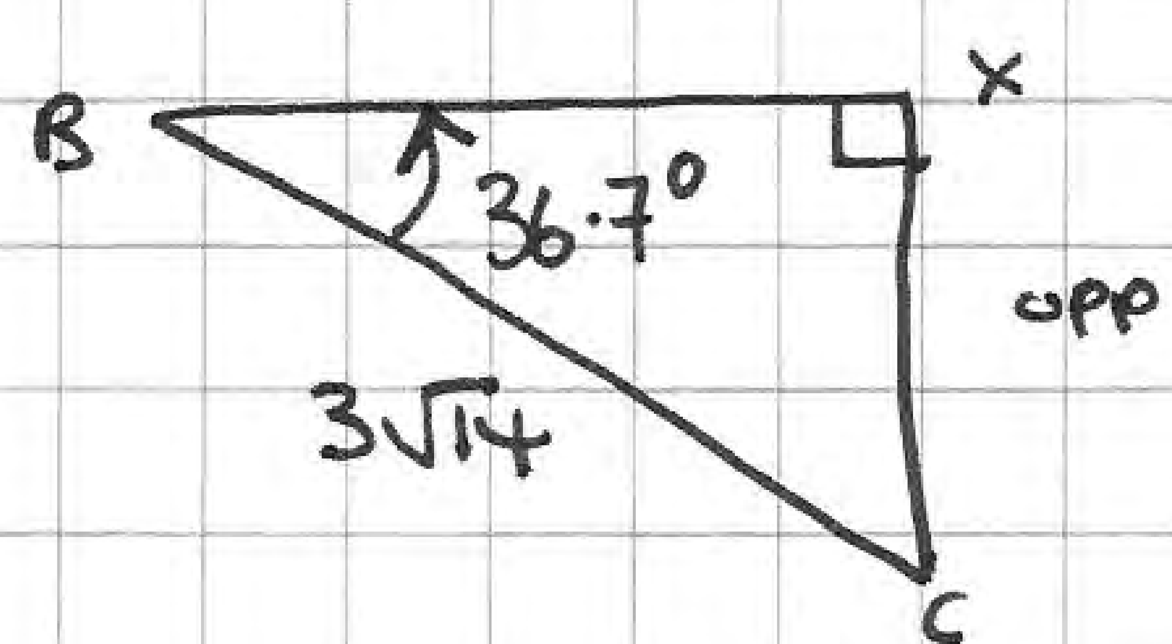
$|\overrightarrow{CB}| = \sqrt{1^2 + 5^2 + 10^2} = \sqrt{126} = 3\sqrt{14}$

c) $\theta = \cos^{-1}\left| \dfrac{\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 5 \\ -10 \end{pmatrix}}{\left|\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}\right| 3\sqrt{14}} \right| = \cos^{-1}\left| \dfrac{2 \times 1 + 1 \times 5 + -2 \times -10}{\sqrt{2^2 + 1^2 + 2^2} \times 3\sqrt{14}} \right|$

$\theta = \cos^{-1}\left(\dfrac{27}{9\sqrt{14}}\right) \Rightarrow \theta = \cos^{-1}\left(\dfrac{3\sqrt{14}}{14}\right) \Rightarrow \theta = 36.7^\circ$

d)




$\text{opp} = 3\sqrt{14}\sin 36.7\ldots = \underline{6.71}$

e) $\text{Area} = \frac{1}{2}(BX)(6.71\ldots) \qquad BX = \sqrt{(3\sqrt{14})^2 - (6.71\ldots)^2} = 9$

$\text{Area} = \frac{1}{2}(9)(6.71\ldots) = \underline{30.2}$

8) $\int \sin^2\theta \, d\theta = \int \frac{1}{2} - \frac{1}{2}\cos 2\theta \, d\theta = \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta + C$

b) $x = \tan\theta \qquad y = 2\sin 2\theta$

$\frac{dx}{d\theta} = \sec^2\theta \qquad \frac{dy}{d\theta} = 4\cos 2\theta$

$\text{Volume} = \pi\int_0^{\frac{1}{\sqrt{3}}} y^2 \, dx = \pi\int_0^{\frac{1}{\sqrt{3}}} (2\sin 2\theta)^2 \frac{dx}{d\theta} \, d\theta$

$x = \frac{1}{\sqrt{3}} \Rightarrow \frac{1}{\sqrt{3}} = \tan\theta \Rightarrow \theta = \frac{\pi}{6}$

$x = 0 \Rightarrow 0 = \tan\theta \Rightarrow \theta = 0$

$\text{Volume} = \pi\int_0^{\frac{\pi}{6}} (4\sin\theta\cos\theta)^2 \sec^2\theta \, d\theta$

$\text{Volume} = 16\pi\int_0^{\frac{\pi}{6}} \sin^2\theta\cos^2\theta \times \frac{1}{\cos^2\theta} \, d\theta = 16\pi\int_0^{\frac{\pi}{6}} \sin^2\theta \, d\theta$ #

c) $16\pi\left[\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{6}}$

$= 16\pi\left[\left(\frac{\pi}{12} - \frac{\sqrt{3}}{8}\right) - (0-0)\right] = \frac{4\pi^2}{3} - 2\pi\sqrt{3}$